Auction-based reservations of network trajectories for autonomous vehicles

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Driven to Discover

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Objective

We propose a system for vehicles to reserve space-time trajectories before departure, with priority in the reservation system determined by an auction. The system provides reduced and known arrival times for high-priority vehicles. Reservations are made through a combinatorial assignment algorithm.

Assumptions

Trajectories are space-time paths with specified arrival times for every spatial point

1. All vehicles reserve trajectories before they depart
2. Vehicles reserve minimum travel time trajectories
3. Cell transmission model for traffic flow
4. First-in-first-out behavior
5. Autonomous intersection management

Trajectory reservation algorithm

1. Iterate through each vehicle in order of priority, allowing it to reserve an available remaining minimum travel time path
2. Update connectivity so that later-reserving vehicles cannot interfere with the newly reserved trajectory

```
Initialize-connectivity()
Sort 𝒱 by b_v descending
for all v ∈ 𝒱 do
    π_v := Shortest-path(r_v, s_v, t_v)
    Reserve(π_v)
end for
```

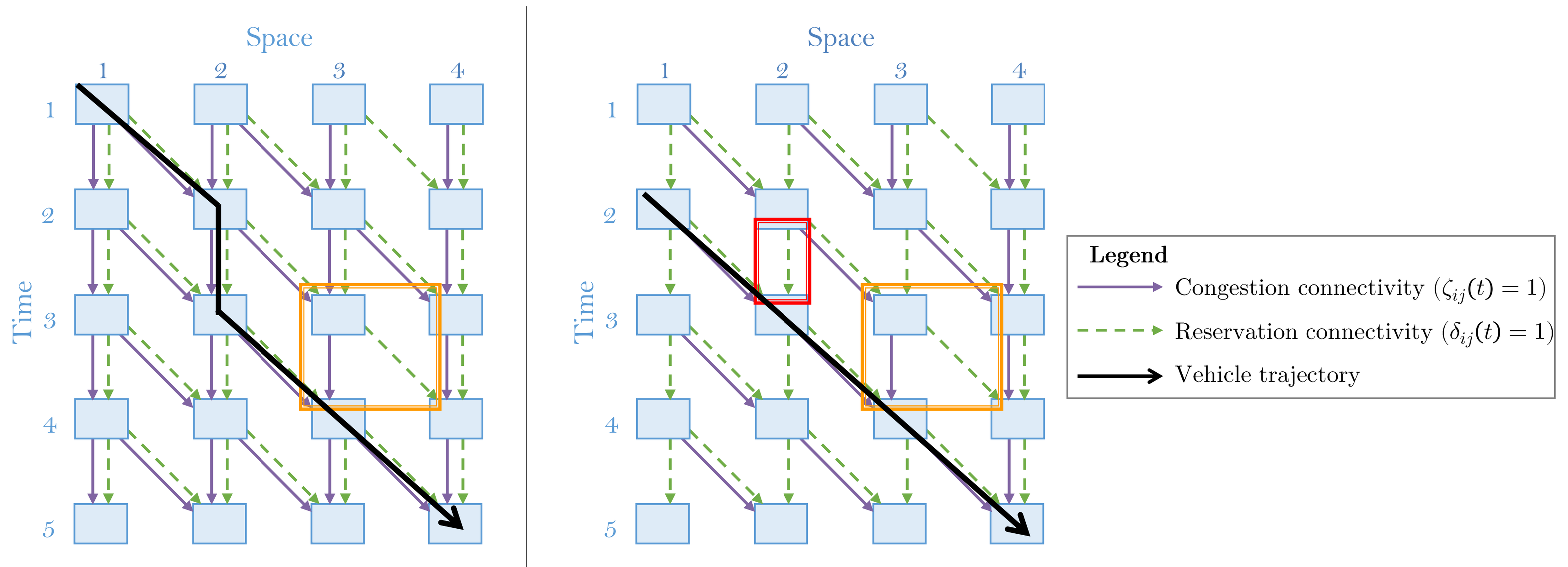
- Preventing vehicles from interfering with already-reserved trajectories is the major challenge with this system

Shortest path

- Construct time-expanded cell network. Cell-time (i, t) is connected to cell-times $(i, t+1)$ and $(i+1, t+1)$.
- Start in cell (r, t); search forward in time until reaching cell s using standard shortest path algorithms (e.g. Dijkstra's)
- Travel from (i, t) to $(j, t+1)$ requires connectivity of $\zeta_{ij}(t) = \delta_{ij}(t) = 1$:
  - $\zeta_{ij}(t)$ is congestion-based connectivity variable
  - $\delta_{ij}(t)$ is reservation-based connectivity variable
- $\delta_{ii}(t) = 0$ **or** $\zeta_{ii}(t)$ **is possible**

Preventing loitering trajectories

Vehicles may find a trajectory with a delay in the middle of a link in violation of traffic flow theory. These trajectories are prevented using congestion connectivity.



(a) (b)

- Setting $\zeta_{22}(2) = 0$ in (b) prevents loitering

Connectivity updates after reserving trajectory

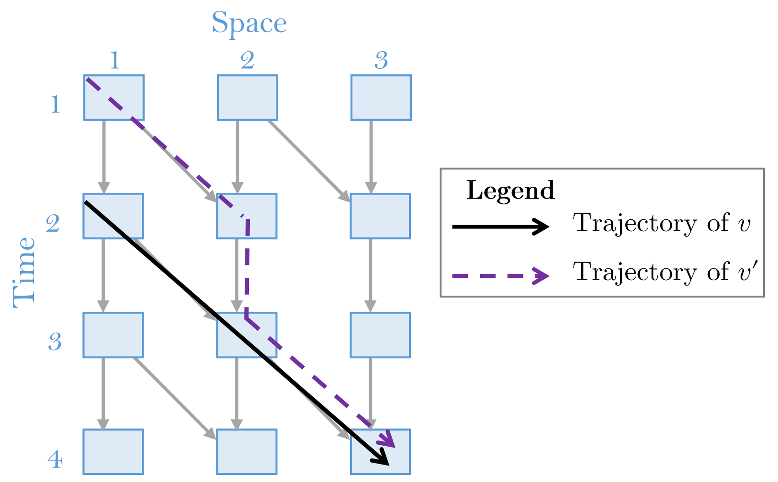
- Set $\delta_{jj}(t-1) = 0$ if
- If $N - \frac{u_j^f}{w_j} \sum_{(i',j) \in \mathcal{E}} y_{i'j}(t) < n_j(t) - 1$ or $n_j(t) + 1 > Q_j$ set $\delta_{jj}(t-1) = 0$
- If $\sum_{(i,j) \in \mathcal{E}/\mathcal{E}^t} y_{ij}(t) + 1 < R_j(t)$ set $\zeta_{ij}(t) = 0$

Initially, $\zeta_{ii}(t) = 0$ unless i is the last cell on a link to prevent congestion in the absence of congested density.

- If $n_j(t) + 1 > Q_j$ or $\sum_{(i,j) \in \mathcal{E}} y_{ij}(t) < R_j(t)$ set $\zeta_{ii}(t) = 1$

FIFO ordering

Earlier-departing, lower-priority vehicle v' could overtake v in FIFO order.



- Solution: After reserving trajectory for v, remove cell connectivity if FIFO would invalidate v's trajectory

Proposition 1

The trajectory reservation algorithm results in vehicle trajectories that satisfy CTM flow constraints and FIFO behavior.

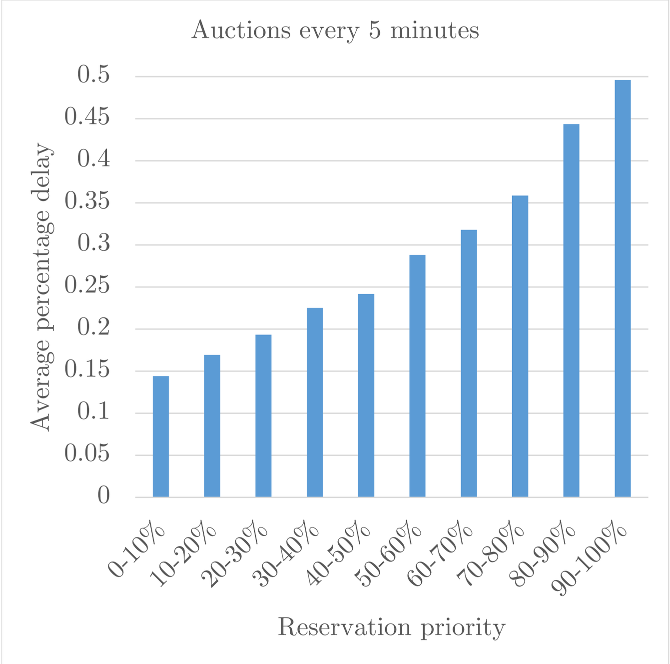
Time complexity

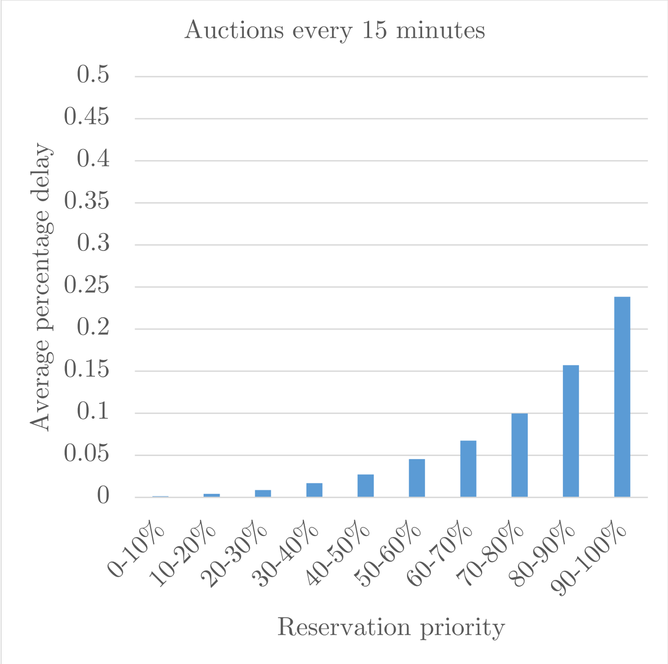
$$O\left(|\mathcal{V}|\left(\log\left(|\mathcal{V}|\right) + |\mathcal{E}|T + |\mathcal{C}|T\log\left(|\mathcal{C}|T\right) + T\Gamma^-\Gamma^+\right)\right)$$

Periodic auctions — Downtown Austin

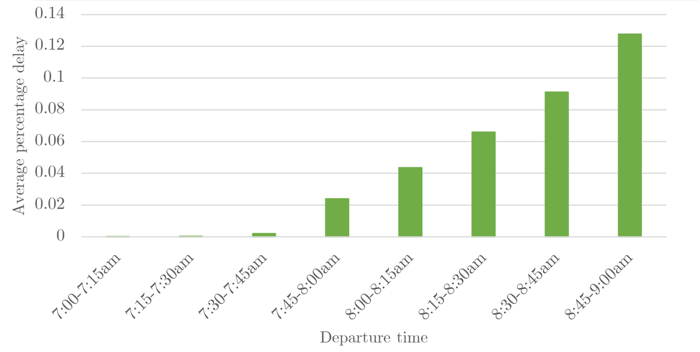
Auctions held every $\mathcal{T}$ minutes.

- More practical; vehicles only need to reserve their trajectory $\mathcal{T}$ minutes before departure

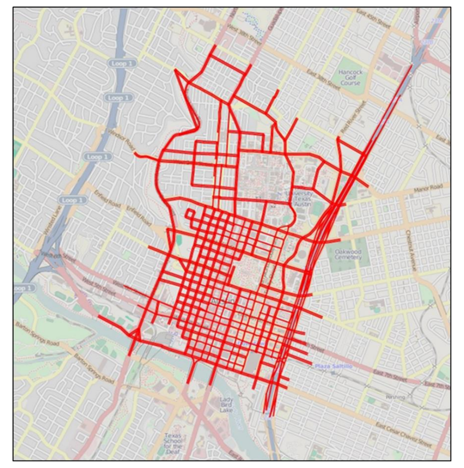




- Benefits to high-bidding travelers are reduced and depend on departure time.

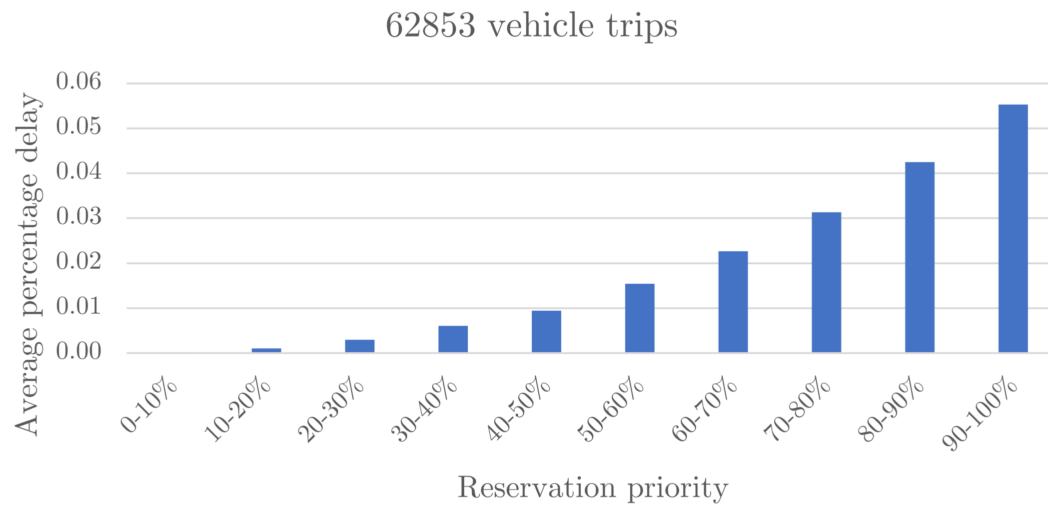


Downtown Austin



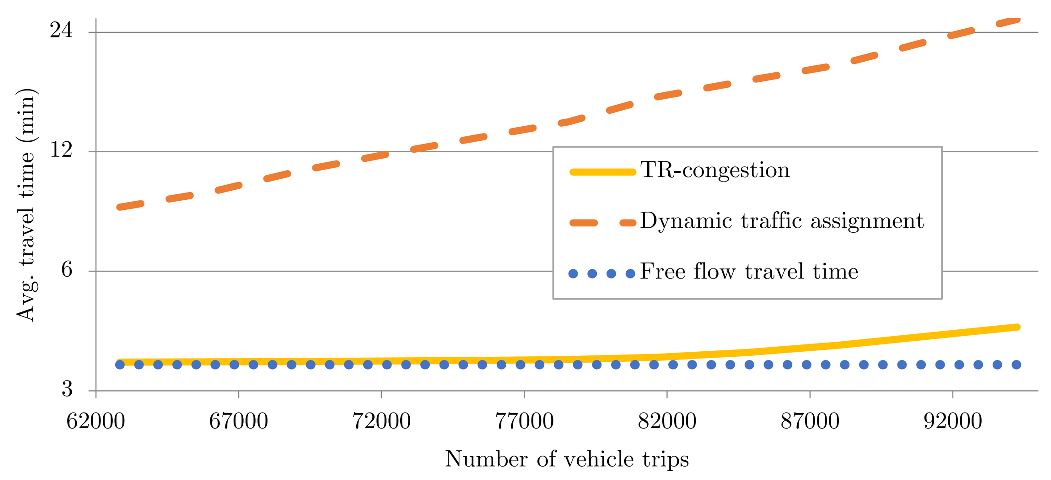
- 634 nodes
- 1574 links
- 3359 cells
- Computation time: 1.95hr
- 1 auction for all vehicles

Travel time and reservation priority



- Lower-priority vehicles experience greater delays on average
- High variance in delays for low-priority vehicles

Average travel time



- Travel times significantly less than dynamic user equilibrium
- Queueing and congestion are reduced as a corollary of maintaining the validity of reserved trajectories

Conclusions

- Combinatorial algorithm for trajectory reservations (compatible with congested networks)
- Reduced travel times for high-priority vehicles
- Lower average travel times for **all** vehicles (compared with dynamic user equilibrium)
- Accepts trips with fixed departure or arrival times

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